Topological Near Homomorphisms

ABSTRACT: Here, some properties related to topological near subgroups are dis- cussed. It is prove that the product of topological near groups is a topological near group. In this chapter, Topological near group homomorphisms are introduced and studied. Finally, near action, near homogenous space and near kernel are defined and studied.

Keywords: Near groups, topological near groups, topological sub neargroups, product of topological near groups, topological near group homomorphisms, topo- logical near homogeneous space, near kernel.

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I.   INTRODUCTION

Rough set, as a mathematical theory for dealing with imprecise, uncertain and in- complete data, was first introduced by pawlak(1982). Its main idea is to use the known incomplete information or knowledge to appriximately describe the concept of imprecise or uncertain, or to deal with ambiguous phenomena and problems ac-cording to the results of observation and measurement. After more than 30 years of research, the theory of rough set has been continuously improved and widely ex- panded in applications, see Wu and Mi (2019). At present, it has been successfully applied in machine learning and knowledge discovery, information system analy- sis, data mining, decision support system, fault detection, process control, pattern recognition, etc.

In [7],Bagirmaz et al. Introduced the concept of topological rough groups they ex- tended the notion of a topological group to include algebraic structures of rough groups. In addition they presented some examples and properties.

In 2002, J.F.Peters developed the near set theory as a genealiztion of rough set the-ory. peters utilized the features of objects to develop the nearness of objects [23] and consequently, the classified our universal, set with respect to the object infor- mation available. The near set approach leads to partitions of ensembles of sample objects with measurable information content and an approach to feature selection. A probe function is a real valued function representing a feature of physical objects such as images or behaviours of individual biological organisms.

The main purpose of this paper is to introduce some basic definitions and results about topological near groups and topological near subgroups. We also introduce the cartesian product of topological near groups.

In this paper, we present near action and near homogenous spaces, and descuss some of their properties we also define a near kernal. We organise the paper as follows, In Scetion 2 we collect the needed material about near groups and near homomor- phisms. Then the definition of topological near groups and importet properties have been recalled in section 3, section 4 presents our main resulits where we introduce, near action and homogenous spaces.

II.   PRELIMINARIES

In this section, some definitions and results about near sets, near groups and topo- logical groups used in this paper are given

Object Description [21]

Objects are known by their description. An object description is defined by means of a tuple of function values $\psi(x)$ associated with an object $x \in X$. The important thing to notice is the choice of functions $\psi_i \in B$ used to describe an object of inter-est.

The intuition underlying a description $\psi(x)$ is recording of measurements from sen- sors, where each sensor is modelled by a function ψ_i.

Table 1: Description Symbols

Symbol	Interpretation
$\Re$	Set of real numbers
O	Set of perceptual objects
X	$X \subseteq O$,set of sample objects
xFB	$x \in O$,sample perceptual object
ψ	A set of functions representing object features,
L	
i	$B \subseteq F$,set of functions representing object features
ψ_i $\psi(x)$	$\psi : O \to \Re^L$, object description
	L is a description length
	$i \leq L$

$\psi_i \in B$, where $\psi_i : X \rightarrow \mathfrak{R}$,probe function,
$\psi(x)=(\psi_1(x),\psi_2(x),\psi_3(x), ,\psi_i(x), , \psi_L(x))$.

Assume that $B \subseteq F$ is a given set of functions representing features of sample objects $X \in O$. Let $\varphi_i \in B$, where $\varphi_i : O \rightarrow \mathfrak{R}$. The value of $\psi_i(x)$ is a measurement associated with a feature of an object $x \in X$. The function ψ_i is called a probe. In combination, the functions representing object features provide a basis for an object description $\psi : O \rightarrow \mathfrak{R}^L$, a vector containing measurements (returned values) associated with each functional value $\psi_i(x)$, where the description length $|\psi| = L$.

Object Description : $\psi(\mathbf{x}) = (\psi_1(\mathbf{x}), \psi_2(\mathbf{x}),....., \psi_i(\mathbf{x}), \psi_L(\mathbf{x}))$

Nearness Objects [21]

Sample objects $X \subseteq O$ are near each other if, and only if the objects have similar descriptions. Recall that each description ψ^1 defines a description of an object. Then let $\Delta\psi_i$ denote

$\Delta\psi_i = \psi_i(x^{'})-\psi_i(x)$,

where $x, x^{'} \in O$. The difference $\Delta\psi$ leads to a definition of the indiscernibility rela- tion $\sim_B$ introduced by Zdzislaw Pawlak [12].

Table 2: Set, Relation, Probe Function Symbols

Symbol	Interpretation
$\sim_B$ $[x]_B$ $O/\sim_B$ ξ_B $\Delta\psi_i$	$\{(x, x^{'}) \mid f(x)= f(x^{'}) \forall f \in B, \}$,indiscernibility relation $[x]_B=\{x \in X \mid x^{'} \sim_B x\}$,elementary granule (class) $O/\sim_B= \{[x]_B \mid x \in O\}$,quotient set Partition $\xi_B = O/\sim_B$ $\Delta\psi_i = \psi_i(x^{'}) - \psi_i(x)$,probe function difference.

Definition 2.1. [21]

Let $X, X^{'} \subseteq O, B \subseteq F$. Set X is near $X^{'}$ if and only if there exists $x \in X, x^{'} \in X^{'}$,

$\psi_i \in B$ such that $x \sim_{\psi i} x^{'}$.

Remark 2.2. [21] If X is near $X^{'}$, then X is a near set relative to $X^{'}$ and $X^{'}$ is a near set relative to X.

Definition 2.3. [21] Let $X \subseteq O$ and $x, x^{'} \in X$. If x is near $x^{'}$, then X is called a near set relative to itself or the reflexive nearness of X.

Definition 2.4. [21] Let $B \subseteq F$ be a set of functions representing features of objects $x, x^{'} \in O$. Objects $x, x^{'}$ are called minimally near each other if there exists $\psi_i \in B$ such that $x \sim_{\{\psi i\}} x^{'}, \Delta_{\psi i} = 0$.

Definition 2.5. [21] Let $x, x^{'} \in O, B \subseteq F$. Then
$\sim_B = \{(x,x^{'}) \in O \times O \mid \forall \psi_i \in B, \Delta_{\psi i} = 0\}$
is called the indiscernibility relation on O,where the description length $i \leq |\psi|$.

Theorem 2.6. [21] The objects in a class $[x]_B \in \xi_B$ are near objects.

Definition 2.7. [3] A topological group is a group (G, $*$) together with a topology on G that satisfies the following two properties:

(1) The mapping f : $G \times G \rightarrow G$ defined by f (x, y) = xy is continuous when G is endowed with the product topology.

(2) The inverse mapping g : $G \rightarrow G$ defined by $g(x) = x^{-1}$ is continuous

We remark that item (1) is equivalent to the statement that, whenever $W \subseteq G$ is open, and $W \in N(x_1x_2)$, then there exists open sets V_1 and V_2 such that $V_1 \in N(x_1); V_2 \in N(x_2)$ and $V_1V_2 = \{x_1x_2/x_1 \in V_1; x_2 \in V_2\} \subseteq W$. Also, item (2) is equivalent to showing that whenever $V \subseteq G$ is open, then $V^{-1} = \{x^{-1} \mid x \in V\} \in N(x^{-1})$ is open. Let G be a topological group and let H be a subgroup of G. Then H becomes a topological group when endowed with the topology induced by G.

Definition 2.8. [12] Let NAS=(O, F, $\sim_{Br}$, N_r, v_{Nr}), be a nearness approximation space and let $\cdot$ be a binary operation defined on O. A subset G of perceptual objects O is called a near group if the following properties are satisfied

(1) $\forall x,y \in G, x \cdot y \in N_r(B)*G$

(2) $\forall x, y, z \in G, (x \cdot y) \cdot z = x \cdot (y \cdot z)$ property holds in $N_r(B)*G$.

(3) $\exists e \in N_r(B)*G$ such that $\forall x \in G, x \cdot e = e \cdot x = x$, e is called the near identityelement of the group G.

(4) $\forall x \in G, \exists y \in G$ such that , $x \cdot y = y \cdot x = e$, y is called the near inverse element of x in G.

Proposition 2.9. [12] Let G be a near group

(1) $\forall x, y \in H, x \cdot y \in N_r(B)^*H$

(2) $\forall x \in H, x^{-1} \in H$

(3) There is one and only one identity element in near group G.

(4) $\forall x \in G$, there is only one y such that $x \cdot y = y \cdot x = e$; we denote it by x^{-1}.(5) $(x^{-1})^{-1} = x$.

(6) $(x \cdot y)^{-1} = y^{-1} \cdot x^{-1}$.

Definition 2.10. [12] Let $G_1 \subset O_1$, $G_2 \subset O_2$ be near groups. If there exists a surjection ϕ : $N_{r1}(B)^*G_1 \to \phi : N_{r2}(B)^*G_2$ such that $\phi(x \cdot y) = \phi(x) \circ \phi(y)$ for all x, y $\in N_{r1}(B)^*G_1$ then ϕ is called a near homomorphism and G_1, G_2 are called near homomorphic groups.

Definition 2.11. [17] A near group G with a topology τ on $N_r(B)^*G$ is called a topological near group if the following hold

(a) f : $G \times G \to N_r(B)^*G$ defined by $f(a, b) = ab$ is continuous with repect to prod-uct topology on G $\times$ G and the topolgy τ_G on G induced by τ.

(b) τ : $G \to G$ defined by $\tau(a) = a^{-1}$ is continuous with respect to the topology τ_G on G induced by τ.

Definition 2.12. [17] Let G be a topological near group. For a fixed element a in G, we define

(i) A mapping L_a : $G \to N_r(B)^*G$ which is defined by $L_a(x) = ax$, is called a left

transformation from G is to $N_r(B)^*G$

(ii) A mapping R_a : $G \to N_r(B)^*G$ which is defined by $R_a(x) = xa$, is called a right

transformation from G is to $N_r(B)^*G$

Definition 2.13. [17] Let G be a topological near group. Then

(a) The left transformation map L_a : $G \to N_r(B)^*G$ is continuous and one - to - one.

(b) The right transformation map R_a : $G \to N_r(B)^*G$ is continuous and one - to -one.

(c) The inverse mapping τ : $G \to G$ is a homeomorphisms for all x $\in$ G

III. CARTESIAN PRODUCT OF TOPOLOGICAL NEARGROUPS

In this section, we discuss some results on cartesian products and introduce near action and near homogenous spaces in topology using near groups.

$(X_1, F_1, \sim_{Br_1}, N_{r1})$ and $(X_2, F_2, \sim_{Br_2}, N_{r2})$ be two nearness approximation spaces and let $*_1$ and $*_2$ be two binary operations on X_1 and X_2 respectively. For x, $x_1 \in X_1$ and y, $y_1 \in X_2$. we have $(x, y)(x^{'}, y^{'}) \in X_1 \times X_2$

Define $*$ as, $(x, y) * (x_1, y_1) = (x *_1 x_1, y *_2 y_1)$

Then $*$ is a binary operation on $X_1 \times X_2$. Indeed that the product of equivalence relation $\sim_{Br_1}$ and

$\sim_{Br_2}$ is also an equivalence relation on $X_1 \times X_2$

Theorem 3.1. Let $G_1 \subseteq X_1$ anf $G_2 \subseteq X_2$ be two near groups. Then the cartesian product $G_1 \times G_2$ is also a near group

For,

(i) $\forall (a_1, b_1), (a_2, b_2) \in G_1 \times G_2$,

$\forall (a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2; b_1 *_2 b_2) \in N_r(B)^*G_1 \times N_r(B)^*G_2$

(ii) Associative law is satisfied forall elements in $N_r(B)^*G_1 \times N_r(B)^*G_2$.

(iii) $\exists$ an identity element (e, $e^{'}$) $\in N_r(B)^*G_1 \times N_r(B)^*G_2$ such that $\forall (x, x^{'}) \in G_1 \times G_2, (x, x^{'}) \times (e, e^{'}) = (e, e^{'}) \times (x, x^{'}) = (ex, e^{'}x^{'}) = (x, x^{'})$

(iv) $\forall (x, x^{'}) \in G_1 \times G_2, \exists$ an element $(y, y^{'}) \in G_1 \times G_2$ such that $(x, x^{'}) * (y, y^{'}) = (x *_1 y, x^{'} *_2 y^{'}) = (y *_1 x, y^{'} *_2 x^{'}) = (y, y^{'}) * (x, x^{'}) = (e, e^{'})$.

Example 3.2. Let X = { 0, 1, 2 } be a set of perceptual objects, B = { ψ_1, ψ_2, ψ_3 } be a set of functions ($*$) be the binary operation, addition modulo 3. Sample valuesnof the probe function {ψ_i} are defined as,

ψ_1 : $X \to V_1$ de f ined by $\psi_1(n) = n(n-1)$ $\forall n \in X$

ψ_2 : $X \to V_2$ de f ined by $\psi_2(n) = n^2$ $\forall n \in X$

ψ_3 : $X \to V_3$ de f ined by $\psi_3(n) = n^2 - n^3$ $\forall n \in X$

	0	1	2
ψ_1	0	0	2
ψ_2	0	1	4
ψ_3	0	0	−4

Let us construct the equivalence classes for each combination, thus equivalence clsses are defined as,

$[0]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(0) = 0\}$,
$= \{0,1\}$
$[2]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(2) = 2\}$,
$= \{2\}$
Hence we have $\xi_{\{\psi_1\}} = \{[0]_{\{\psi_1\}}, [2]_{\{\psi_1\}}\}$

$[0]_{\{\psi_2\}} = \{x' \in X | \psi_2(x') = \psi_2(0) = 0\}$,
$= \{0\}$
$[1]_{\{\psi_2\}} = \{x' \in X | \psi_2(x') = \psi_2(1) = 1\}$,
$= \{1\}$
$[2]_{\{\psi_2\}} = \{x' \in X | \psi_2(x') = \psi_2(2) = 4\}$,
$= \{2\}$
Hence we have $\xi_{\{\psi_2\}} = \{[0]_{\{\psi_2\}}, [1]_{\{\psi_2\}}, [2]_{\{\psi_2\}}\}$

$[0]_{\{\psi_3\}} = \{x' \in X | \psi_3(x') = \psi_3(0) = 0\}$,
$= \{0\}$
$[2]_{\{\psi_3\}} = \{x' \in X | \psi_3(x') = \psi_3(2) = -4\}$,
$= \{2\}$
Hence we have $\xi_{\{\psi_3\}} = \{[0]_{\{\psi_3\}}, [2]_{\{\psi_3\}}\}$
Therefore, for r = 1, a classification of X is $N_1(B) = \{\xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}}\}$
Let G = { 1,2 } be a subset of the perceptual objects,
$N_r(B)^* G = \bigcup\limits_{x:[x]_{\{\psi_i\}} \cap G \neq \emptyset} [x]_{\{\psi_i\}}$

$= \{\{0,1\} \cup \{2\} \cup \{1\} \cup \{2\} \cup \{0,1\} \cup \{2\}\}$
$= \{0,1,2\}$
The cartesian product $X \times X$ as follows
$X \times X = \{(0,0),(2,2),(1,1),(0,1),(0,2),(1,2),(2,1),(1,0),(2,0)\}$

Then the new classification is, $\{\{[0]_{\psi_1}, [2]_{\psi_1}\}, \{[0]_{\psi_2},[1]_{\psi_2},[2]_{\psi_2}\}, \{[0]_{\psi_3},[2]_{\psi_3}\}\}$ Consider the near group G = { 1,2 } then the cartesian product $G \times G$ is
$G \times G = \{(1,1)(1,2)(2,2)(2,1)\}$
where $N_r(B)^*(G \times G) = N_r(B)^*G \times N_r(B)^*G = X \times X$. From the definition of a neargroup, we have that,
(i) The multiplication of elements in $G \times G$ is closed under $N_r(B)^*G \times N_r(B)^*G$, where
$*_1$ and $*_2$ are
$(1,1)*(1,2) = (2,0)$
$(1,1)*(2,2) = (0,0)$
$(1,1)*(2,1) = (0,2)$
$(1,2)*(1,1) = (2,0)$
$(1,2)*(2,2) = (0,1)$
$(1,2)*(2,1) = (0,0)$

$(2,2)*(1,1) = (0,0)$
$(2,2)*(1,2) = (0,1)$
$(2,2)*(2,1) = (1,0)$
$(2,1)*(1,1) = (0,2)$
$(2,1)*(1,2) = (0,0)$
$(2,1)*(2,2) = (1,0)$

(ii) The associative law is satisfied.
(iii) There exists $(0,0) \in N_r(B)^*G \times N_r(B)^*G$
such that for every $(g, g') \in G \times G$, we have $(0,0) * (g, g') = (g,g')$
(iv) For every element of $G \times G$, $\exists$ an inverse element in $G \times G$, where
$(1,1)^{-1} = (2,2) \in G \times G$,
$(2,1)^{-1} = (1,2) \in G \times G$
$(1,2)^{-1} = (2,1) \in G \times G$
$(2,2)^{-1} = (1,1) \in G \times G$
Hence $G \times G$ is a near group.

Example 3.3. Let U = { 0,1,2 } be a set of perceptual objects, B = { ψ_1, ψ_2, ψ_3 } be a set of functions ($*$) be the binary operation, addition modulo 3. Sample values of the probe function $\{\psi_i\}$ are defined as,
ψ_1 : $U \to V_1$ defined by $\psi_1(n) = n(n-1)$ $\forall n \in U$
ψ_2 : $U \to V_2$ defined by $\psi_2(n) = n(n-1)(n-2)$ $\forall n \in U$
ψ_3 : $U \to V_3$ defined by $\psi_3(n) = n(n-1)(n-2)(n-3)$ $\forall n \in U$

	0	1	2
ψ_1	0	0	2
ψ_2	0	0	0
ψ_3	0	0	0

Let us construct the equivalence classes for each combination, thus equivalence clsses are defined as,
$[0]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(0) = 0\}$,
$= \{0,1\}$
$[2]_{\{\psi_1\}} = \{x' \in X | \psi_1(x') = \psi_1(2) = 2\}$,
$= \{2\}$
Hence we have $\xi_{\{\psi_1\}} = \{[0]_{\{\psi_1\}}, [2]_{\{\psi_1\}}\}$
$[0]_{\{\psi_2\}} = \{x' \in X | \psi_2(x') = \psi_2(0) = 0\}$,

= { 0,1,2 }

Hence we have $\xi_{\{\psi_2\}} = \{[0]\{\psi_2\}\}$

$[0]\{\psi_3\} = \{ x' \in X | \psi_3(x') = \psi_3(0) = 0 \}$,

= { 0,1,2 }

Hence we have $\xi_{\{\psi_3\}} = \{[0]\{\psi_3\}\}$ Therefore, for

r = 1, a classification of X is $N_1(B) = \{ \xi_{\{\psi_1\}}, \xi_{\{\psi_2\}}, \xi_{\{\psi_3\}} \}$

Let G = { 1,2 } be a subset of the perceptual objects,

$$N_r(B)^* G = \overset{S}{\underset{x:[x]_{\psi_i} \cap G \neq \emptyset}{}} [x]\{\psi_i\}$$

= { { 0,1 } ∪ { 2 } ∪ { 0,1,2 } ∪ { 0,1,2 } }

= { 0,1,2 }

From Definition

(1) $\forall$ a,b $\in$ G, ab $\in$ $N_r(B)^*$ (G)

(2) The Property $\forall$ a,b,c $\in$ G, (a · b) · c = a · (b · c) holds in $N_r(B)'$ (G)

(3) $\exists$ 0 $\in$ $N_r(B)'$ (G) such that $\forall$ a $\in$ G, a · 0 = 0 · a = a

(4) $\forall$ a $\in$ G, $\exists$ b $\in$ G such that a · b = b · a = e (b is called a near inverse of a in G)G is a near group

Let τ = { $\emptyset$, $N_r(B)'$ (G), { 1 }, { 2 }, { 1, 2 } } on $N_r(B)'$ (G)

Then, τ_G = { $\emptyset$, G, { 1 }, { 2 } } is the relative topology on G From Def (topological near group)

(a) 1 ∗ 1 = 2, for T $\in$ N(2) $\subseteq$ τ, there exist open set U = { 1 } $\in$ N(1) $\subseteq$ τ_G, such that UU $\subseteq$ T

1 ∗ 2 = 0, for T $\in$ N(0) $\subseteq$ τ, there exist open set U = { 1 } $\in$ N(1) $\subseteq$ τ_G, and

V = { 2 } $\in$ N(2) $\subseteq$ τ_G such that UV $\subseteq$ T

2 ∗ 2 = 1, for T $\in$ N(1) $\subseteq$ τ, there exist open set U = { 2 } $\in$ N(2) $\subseteq$ τ_G, such that UU $\subseteq$ T

(b) $\{1\}^{-1}$ = { 2 } is open

$\{2\}^{-1}$ = { 1 } is open

Therefore G is a topological near group.

Hence the product of topological near group we have

τ = { $\emptyset$, $N_r(B)'$ G, { 1 }, { 2 }, { 1, 2 } } as a topology on $N_r(B)'$ G.

then $\tau \times \tau$ is the product topology of $N_r(B)^*$ G $\times$ $N_r(B)^*$ G. Also we have

τ_G = { $\emptyset$, G, { 1 }, { 2 }, { 1, 2 } } as a relative topology on G, then $\tau_G \times \tau_G$ a product topology on G×G is induced by $\tau \times \tau$.

Therefore consider the multiplication map

f : (G × G)×(G × G) → $N_r(B)^*$ G ×$N_r(B)^*$ G. This map is continuous with respect

to the topology $\tau \times \tau$ and product topology on (G × G)×(G × G). Now, consider the inverse map τ : G × G → G × G. This map is continuous. Hence G × G is

a topological near group.

IV. NEAR ACTION AND NEAR HOMEOGENOUS SPACES IN CLASSICAL SET TOPOLOGY

This section deals with near action and near homeogenous spaces in classical set topology.

Let $(X_1, F_1, \sim_{Br_1}, N_{r_1}, v_{Nr_1})$ and $(X_2, F_2, \sim_{Br_2}, N_{r_2}, v_{Nr_2})$ be a nearness approxima-

tion spaces. Let $G_1 \subseteq X_1$ and $G_2 \subseteq X_2$ be topological near groups such that τ_1 and τ_2 are topologies on $N_r(B)^* G_1$ and $N_r(B)^* G_2$, respectively inducing τ_{G1} and τ_{G2} on G_1 and G_2 respectively,

A mapping f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is called a topological near group homomor- phism, if f is a near homomorphism and continuous with respect to the topology τ_2 on $N_r(B)^* G_2$ inducing τ_{G2} on G_2 and the topology τ_1 on $N_r(B)^* G_1$ inducing τ_{G1} on G_1.

A topological near group homomorphism f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is called a topological near group homeomorphism f^{-1} : $N_r(B)^* G_2 \to N_r(B)^* G_1$ such that $f^{-1} \circ f = 1_{N_r(B)^* G_1}$.

Let $(X, F, \sim_{Br}, N_r, v_{Nr})$ be an approximation space. Assume that, G and X are

two subsets of U such that G is a topological near group, and X is a topological near space inducing the topology near space X i.e. near set with ordinary topology. Then we are ready to give the definition of the action of a near group G on a near space is given.

Definition 4.1. Let $(X_1, F_1, \sim_{Br_1}, N_{r_1}, v_{Nr_1})$ and $(X_2, F_2, \sim_{Br_2}, N_{r_2}, v_{Nr_2})$ be near- ness approximation spaces and $\ast_1$, $\ast_2$ be binary operations on X_1 and X_2 respec- tively, let $G_1 \subseteq X_1$ and $G_2 \subseteq X_2$ be two near groups. If the mapping

f : $N_r(B)' G_1 \to N_r(B)' G_2$ satisfies that f (x $\ast_1$ y) = f(x) $\ast_2$ f(y), for all x,y $\in$ $N_r(B)^* G_1$ then f is called a near homomorphism.

Definition 4.2. Let G_1 and G_2 be two near groups. A near homomorphism f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is said to be :

(a) a near epimorphism if f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is onto

(b) a near monomorphism if f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is one - to- one

(c) a near isomorphism if f : $N_r(B)^* G_1 \to N_r(B)^* G_2$ is both onto and one-to-one.

Definition 4.3. A continuous map $f : N_r(B)^* G \times X \to X$ (resp $f : X \times N_r(B)^* G \to X$) is called a left (resp right) near action of G on X, if it satisfies the following con- ditions.

(a) $g(g'x) = (gg')x$ (resp. $((xg)g') = x(gg'))$, for all $g,g' \in N_r(B)^* G$ and $x \in X$

(b) $ex = x$ (resp. $xe = x$), for every $x \in X$, — where $e \in N_r(B)^* G$ is the near identity. Then the near set X is called a near G- space.

The action f is said to be effective if $gx = g'x$, for every $x \in X \Rightarrow g = g'$. In addition the action f is said to be transitive if for every x, $x' \in X$ there exists $g \in N_r(B)^* G$ such that $gx = x'$

Definition 4.4. Let X be a near G- space. Then X is said to be topologically near homogeneous if for all x, $y \in X$, there is a topological homeomorphism $f : X \to X$ such that $f(x) = y$.

Proposition 4.5. Let G be a topological near group and X be a near G-space. Then the left transformation map $L_g : X \to X$ (resp right transformation map $R_g : X \to X$), for every $g \in$ — G,which is defined $L_g(x) = gx$ $(R_g(x) = xg)$, is a topological homeomorphism.

Proof. The continuity of the action f implies the continuity of L_g. The continuous
(a) and (b) in Definition 4.3 are respectively equivalent to

(i) $L_g \circ L_g' = L_{gg}'$

(ii) $L_e = 1_X$. —

Therefore , the maps L_g and L_g^{-1} are inverse of each other. Thus, L_g is a topological homeomorphism from X to X.

Note that, the left (resp.right) transformation map $L_g(R_g) : X \to X$,is not topolog- ically homomorphism for every $g \in N_r(B)^* G$. This is true only in the case that $N_r(B)^* G$ is a group.

Corollary 4.6. Let G be topological near group. Then for every open set O in X and $g \in G$, $L_g(O) = gO$ is open in X

Proof. By Theorem 4.5, $L_g : X \to X$ is a topological homeomorphism. Thus $L_g(A) = gO$ is a open set in X

Theorem 4.7. Let G be a topological near group such that $N_r(B)^* G$ is a group. For any open subset O of $N_r(B)^* G$, if A is a subset of $N_r(B)^* G$, then AO(respectively OA) is open in $N_r(B)^* G$.

Proof. In actual fact $N_r(B)^* G$ is a group implies G acts on itself. Thus for ev- ery $g \in N_r(B)^* G$, L_g is a topological homeomorphism. The rest of proof follows immediately from left (right) transformation. Because that $AO = _{a \in A} L_a(O)$ and $OA = _{a \in A} R_a(Q)$. is open in $N_r(B)^* G$.

Theorem 4.8. Let G be an topological near group such that $N_r(B)^* G$ is a group. Let H be a sub neargroup of G such that $N_r(B)^* H$ is closed under multiplication. If there is an open set O in G such that $e \in O$ and $O \subseteq H$, then $N_r(B)^* H$ is an open set in $N_r(B)^* G$.

Proof. Let O be a non-empty open set in G such that $O \subseteq H$ and $e \in O$. Then for every $h \in N_r(B)^* H, L_h(O) = hO$ is open in $N_r(B)^* G$. Hence $N_r(B)^* H = _{h \in N_r(B)^* G} hO$ is open in $N_r(B)^* G$.

Theorem 4.9. Let G be a topological near group such that $N_r(B)^* G$ is a group and let H be a sub neargroup of G. Let H be a sub neargroup of G. Let O be an open set in G such that $O \subseteq H$. Then for every $h \in H$,hO is an open set in $N_r(B)^* H$

Proof. Since $N_r(B)^* H \subseteq N_r(B)^* G$ and $N_r(B)^* G$ is a group, L_h is a topological homeomorphism. By the definition of left transformation, $L_h(O) = hO$ is open in $N_r(B)^* G$ the fact that $O \subseteq H$ implies $hO \in N_r(B)^* H$. Hence, hO is open in $N_r(B)^* H$.

Definition 4.10. Let G_1 and G_2 be topological near groups, $f : N_r(B)^* G_1 \to N_r(B)^* G_2$ be a topological near group homomorphism and let e_2 be the near iden-tity element of G_2. Then
$Ker(f) = \{ g \in N_r(B)^* G_1 : f(g) = e_2 \}$ is called the near kernal associated to the map f.

Theorem 4.11. Let f be a topological near group homomorphism from $N_r(B)^* G_1$ to $N_r(B)^* G_2$. Then the near kernal is a normal sub neargroup of $N_r(B)^* G_1$.

Proof. Let $*$ and $*_1$ be the binary operation in G_1 and G_2 respectively. Since
$f(e_1) = e_2$, $e_1 \in ker(f)$. ker(f)
and $f(y) = e_2$
0/ For every $x,y \in ker(f)$, we have $f(x) = e_2$

(a) Since $f(x * y) = f(x) *_1 f(y) = e_2$, we have $x * y \in ker(f)$
(b) Also $f(x^{-1}) = (f(x))^{-1} = (e_2)^{-1}$.
Hence ker(f) is a sub neargroup of G_1
(c)   For every $x \in G_1$ and $r \in ker(f)$, we have $f(x * r * x^{-1}) = f(x) *_1 f(r) *_1 f(x^{-1}) = f(x) *_1 e_2 *_1 f(x^{-1}) = f(x) *_1 (f(x))^{-1} = e_2$. Therefore, $x * r * x^{-1} \in ker(f)$ thus ker(f) is a normal sub neargroup of G_1. □

Example 4.12. Consider the map $f : N_r(B)^* G_U \to N_r(B)^* G_X$, where G_1 and G_2 are near groups as above respectively,
Define f as follows,
$f(0) = 0, f(1) = 0, f(2) = 0$
clearly, f is a continuous and homomorphism. Hence f is a topological near group homomorphism from Def 4.10, it is easy to see that $ker(f) = \{0,1,2\}$

is a subset of
$N_r(B)^* G_U$. Moreover ker(f) is a normal sub neargroup of $N_r(B)^* G_U$.

Definition 4.13. Let G be a topological near group and $B \subseteq \tau$ be a base for τ.
For $g \in G$, the family

$B_g = \{O \cap G : O \in B, g \in O\} \subseteq B$ is called a base at g in τ_G.

Theorem 4.14. Let G be a topological near group such that the identity ele- ment $e \in G$ and $N_r(B)^* G$ is Closed under mulitiplication. Let G be an open set in $N_r(B)^* G$. For $g \in G$ the base of g in $N_r(B)^* G$ is equal to

$B_g = \{gO : O \in B_e\}$
where B_e is the base of the identity e in τ_G

Proof. Since $g \in G$, we have $g \in N_r(B)^* G$. Let O_1 be an open set in $N_r(B)^* G$ and let $g \in O_1$. Since $e \in G$, and G is a topological near group, there are two open sets O_2 and O_3 such that $g \in O_2$, $e \in O_3$ and $f(O_2 \times O_3) \subseteq O_1$. We have G is an open set in τ. Then O_3 is a neighbourhood of e in τ. Then there is a basic open set $O \in B_e$ such that $e \in O \subseteq O_3$. Hence $L_g(O) = gO \subseteq f(O_2 \times O) \subseteq f(O_2 \times O_3) \subseteq O_1$.

Definition 4.15. Let G be a neargroup and $A \subset G$. Then A isnsymmetric if A =
A^{-1}

Proposition 4.16. Let G be a topological near group. if $e \in G$, then for each open neighbourhood O of e in G, there exists a symmetric open neighbourhood P ofe in G such that $P^2 \cap G \subset O$
Proof. Take an arbitrary open neighbourhood O of e in G. Then there exists an open neighbourhood W of e in $N_r(B)^* G$ such that $O = W \cap G$. Since $f : G \times G \rightarrow N_r(B)^* G$ is continuous at point (e, e) and the inverse mapping is a homeomorphism, there exists a symmetric open neighbourhood P of e in G such that $P^2 \subset W$:; Hence $P^2 \cap G \subset O$.

Proposition 4.17. Let f be a near homomorphism between near groups G_1 and G_2. If G_1 is a topological group, then G_2 is also a topological group.
Proof. It suffices to prove that $G^2 = G_2$. Take arbitrary x, y $\in G_2$. Then, there eists g, h $\in G_1$ such that f (g) = x, f (h) = y hence, f (gh) = f (g) f (h) = xy. Since G_1 is a topological group, it follows that gh $\in G_1$; thus f (gh) $\in G_2$. i.e xy $\in G_2$. Therefore, G_2 is a topological group.

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